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புனித பேதுரு கல்லூரி - கொழும்பு 04
St. Peter's College - Colombo 04

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First Term Examination - 2024

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GRADE 12

ixhqla; .Ks;h

COMBINED MATHEMATICS

$2\frac{1}{2}$ meh
Hour

- **Answer 05 questions only.**

01. a). Express as a rational number,

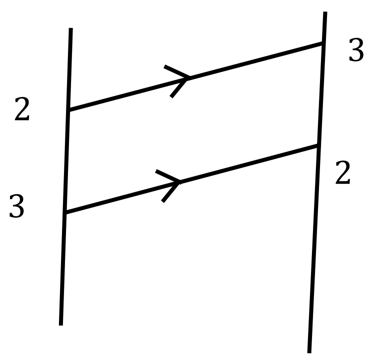
$$\mathbf{0.1\dot{7} + 2.\dot{1}\dot{4}}$$

b). Simplify, $\dfrac{x}{y^{1/2} + x^{1/2}} + \dfrac{x}{y^{1/2} - x^{1/2}}$

c). Solve for x, $\dfrac{\sqrt{5 - x^2}}{x + 1} = 1$

02. a). The arrow diagram shows a part of mapping

$$x \rightarrow \frac{a}{x - 1} + b$$



$f : x \rightarrow \dfrac{a}{(x - 1)} + b$

Find,

i). The value of a and b

ii). $f(O)$

iii). $f(x + k)$

b). Find the domain and range of the following functions.

i). $\boldsymbol{f : x \to \frac{1}{2}x + 3}$

ii). $\boldsymbol{f : x \to \frac{1}{x-2}}$

iii). $\boldsymbol{f : x \to \sqrt{x-4}}$

c). If $\boldsymbol{f : x \to 3x - 1}$ and $\boldsymbol{g : x \to \frac{3}{x-4}}$ then find,

i). $\boldsymbol{f^{-1}(-4)}$ ii). $\boldsymbol{g^{-1}(3)}$ iii). $\boldsymbol{gf(x)}$ iv). $\boldsymbol{gg(x)}$

03. a). State the remainder theorem and prove it. Show that when a polynomial $\boldsymbol{f(x)}$ is divided by $(\boldsymbol{Px - q})$, Where $\boldsymbol{\rho \neq O}$, the remainder is $\boldsymbol{f\left(\frac{q}{p}\right)}$.

If $\boldsymbol{f(x) = 2x^4 + ax^3 + bx + 1}$, Where $\boldsymbol{a}$ and $\boldsymbol{b}$ are real constants. Given that $\boldsymbol{f\left(-{}^{1}/_{2}\right) = 0}$

$\boldsymbol{f(-2) = 21}$. Find $\boldsymbol{a}$ and $\boldsymbol{b}$

Find the two real linear factors of $\boldsymbol{f(x)}$

b. Express in partial fractions

i). $\boldsymbol{\frac{x+2}{(x-3)(x+1)}}$ ii). $\boldsymbol{\frac{x^3}{(x-1)^2(x+2)}}$

04. a). If $\underline{\boldsymbol{a}}$ and $\underline{\boldsymbol{b}}$ are non-zero and non-parallel vectors and $\boldsymbol{\lambda\underline{a} + \mu\underline{b} = 0}$, then prove that, $\boldsymbol{\lambda = 0}$ and $\boldsymbol{\mu = 0}$.

Let $\underline{\boldsymbol{a}} = \boldsymbol{\lambda\underline{i} + 3\underline{i}}$ and $\underline{\boldsymbol{b}} = \boldsymbol{7\underline{i} + \underline{j}}$ where $\underline{\boldsymbol{i}}$ and $\underline{\boldsymbol{j}}$ are unit vectors along $\boldsymbol{ox}$ and $\boldsymbol{oy}$ axis. If $\underline{\boldsymbol{a}}$ and $\underline{\boldsymbol{b}}$ are parallel vectors, then find the value of $\boldsymbol{\lambda}$.

b). If the position vectors of the points $\boldsymbol{A}$ and $\boldsymbol{B}$ are $\boldsymbol{\underline{i} - 3\underline{j}}$ and $\boldsymbol{2\underline{i} + 5\underline{j}}$ respectively. Find,

i). $|\overrightarrow{\boldsymbol{AB}}|$

ii). the unit vector along $\overrightarrow{\boldsymbol{AB}}$

iii). The position vector of the point, which divided $\boldsymbol{AB}$ in the ratio $\mathbf{1:2}$ internally.

c). Define the scalar product of the vectors $\underline{\boldsymbol{a}}$ and $\underline{\boldsymbol{b}}$

Given that two vectors $\underline{\boldsymbol{a}}$ and $\underline{\boldsymbol{b}}$, where $\boldsymbol{a} \neq \mathbf{0}$ and $\boldsymbol{b} \neq \mathbf{0}$. If $(\underline{\boldsymbol{a}} + \underline{\boldsymbol{b}})$ and $(\underline{\boldsymbol{a}} - \underline{\boldsymbol{b}})$ are perpendicular then show that $|\underline{\boldsymbol{a}}| = |\underline{\boldsymbol{b}}|$

05. Prove the following identities.

a). $$\frac{\boldsymbol{Sin\ A + Sin\ 3A}}{\boldsymbol{Cos\ A + Cos\ 3A}} = \mathbf{tan}\ \mathbf{2}\boldsymbol{A}$$

b). $$\boldsymbol{Cos^4\ A + Sin^4\ A = 1 - \frac{1}{2}\ Sin^2\ 2A}$$

c). $$\boldsymbol{Cos\ (A+B) + Sin\ (A-B) = 2\ Sin\ (45+A).Cos\ (45° + B)}$$

d). $$\boldsymbol{Sec\ A = \tan A\ .\ tan\ {}^{A}/_{2} + 1}$$

06. a). Show that If $\boldsymbol{Sin\ \theta = -{}^{1}/_{3}}$ and $\boldsymbol{\pi < \theta < 3\ {}^{\pi}/_{2}}$

then $\boldsymbol{Sin\ 2\theta = \frac{4\sqrt{2}}{9}}$ and $\boldsymbol{\tan 2\theta = \frac{4\sqrt{2}}{7}}$

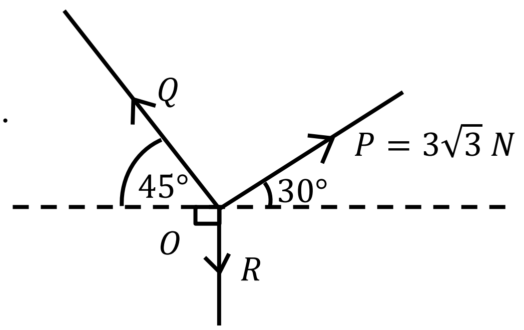
b). If $\boldsymbol{ABC}$ is a triangle, show that

$$\boldsymbol{Cos\ A + Cos\ B - Cos\ C = -1 + 4\ Cos\ {}^{A}/_{2}\ .Cos\ {}^{B}/_{2}\ Sin\ {}^{C}/_{2}}$$

c). Find the general solution of the following equation $\boldsymbol{\sqrt{3}\ Sin\ x - Cos\ x = \sqrt{2}}$.

07.

a).



Three coplanar $\boldsymbol{P}$, $\boldsymbol{Q}$ and $\boldsymbol{R}$ act at $\boldsymbol{O}$ and the system is in equilibrium. Find the values of $\boldsymbol{Q}$ and $\boldsymbol{R}$.

b). The resultant of $\boldsymbol{P}$ and $\boldsymbol{Q}$ is $\boldsymbol{R}$. If $\boldsymbol{P}$ is reversed and $\boldsymbol{Q}$ remaining the same, then the resultant is $\boldsymbol{R^1}$. If $\boldsymbol{R}$ and $\boldsymbol{R^1}$ are perpendicular, then show that, $\boldsymbol{P = Q}$.

c). The resultant of two forces F and $2F$ is perpendicular to the force F. Find the angle between the two forces and the magnitude of the resultant.